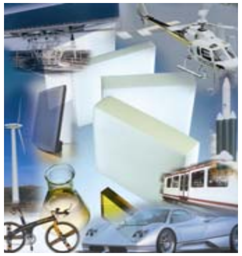


ROHACELL®

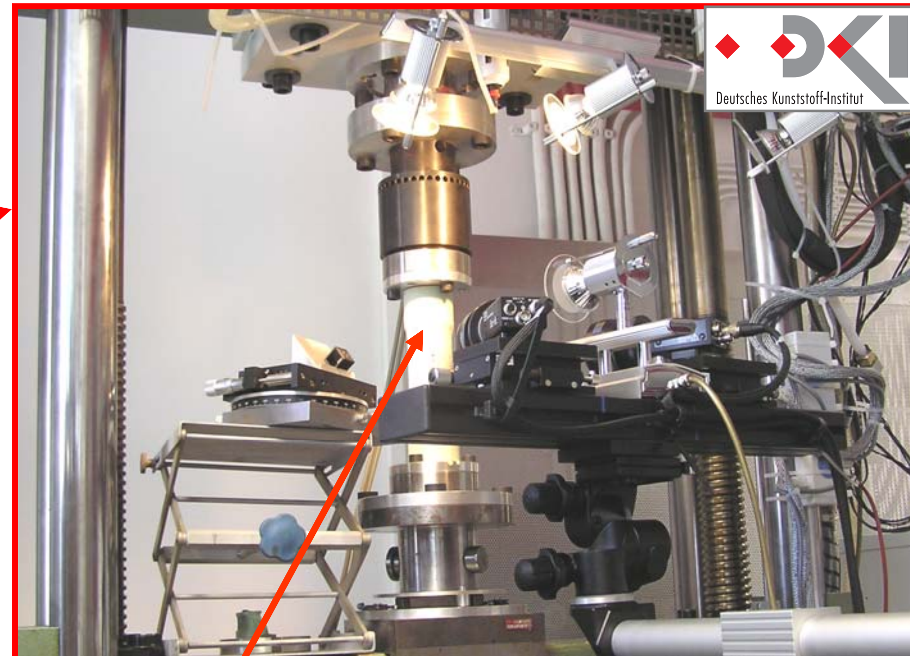
The Core for Sandwich Solutions

Engineering Services:

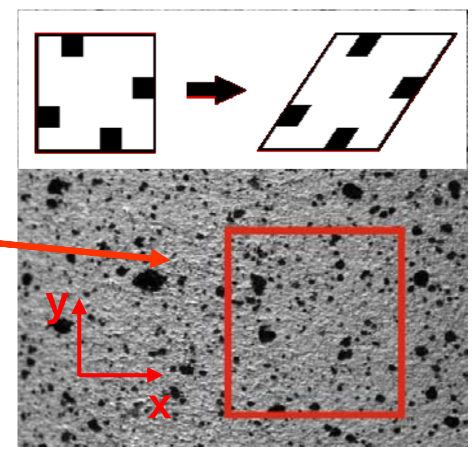
3-D Failure Criterion

EVONIK INDUSTRIES

Test Facility: Tension-/Compression-/Torsion-Testmachine

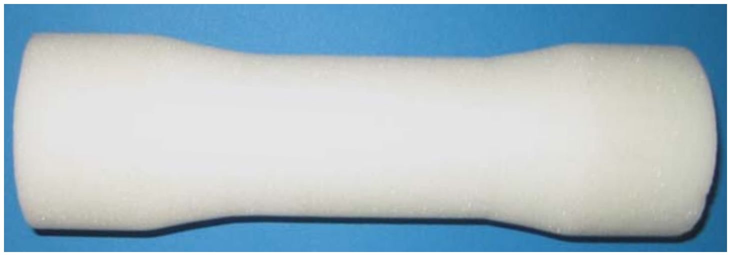


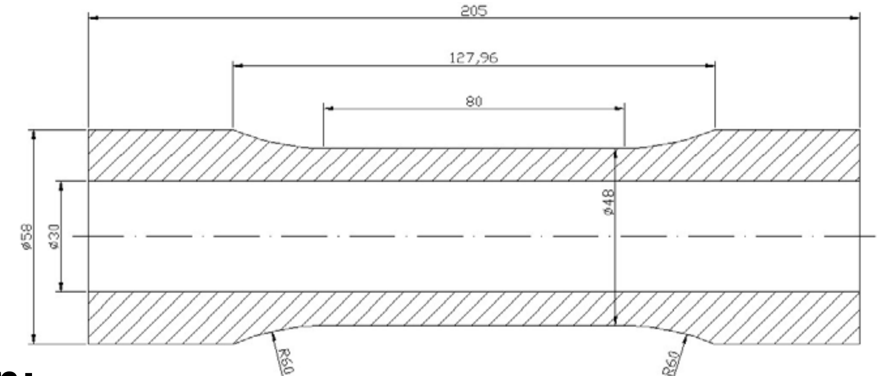
- **Using a Tension-/Compression-/Torsion-Test-Machine**
- **Using an Optical Deformation Measurement System ARAMIS**
- **Using Tubular Test Specimens made of ROHACELL® 51 WF, 110 WF and 200 WF**



Test Facility: Tension-/Compression-/Torsion-Testmachine

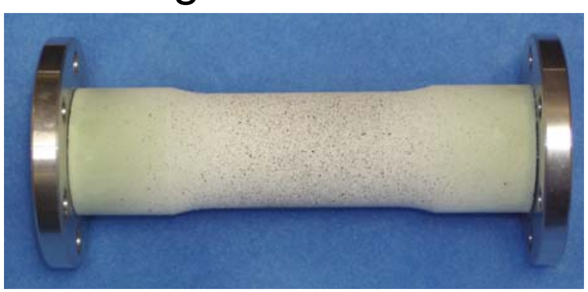
Geometry of the Test Specimen:

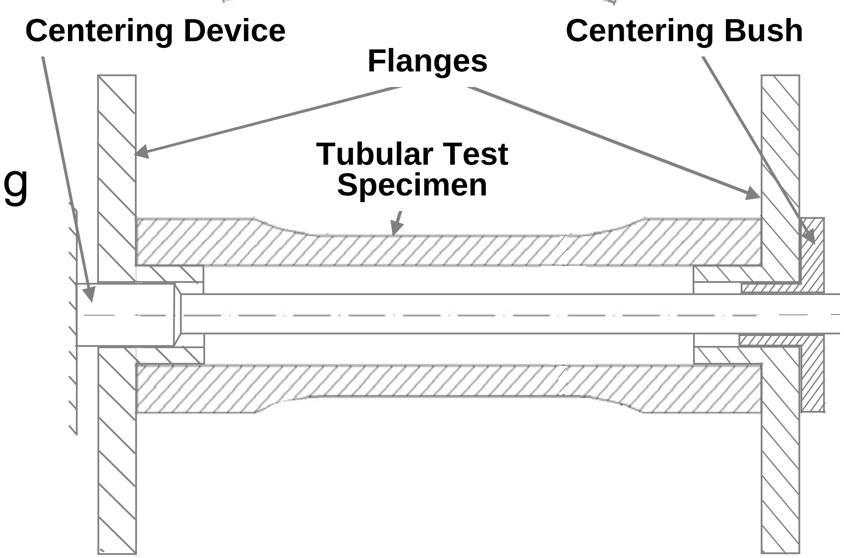




Preparation of the Test Specimen:

- Tubular Test Specimens are glued on Flanges
- Test Specimens are assembled using coaxial Centering Device





Tested ROHACELL® Grades und Types:

- 51, 110, 200 WF
- 71 RIST
- 21 HP

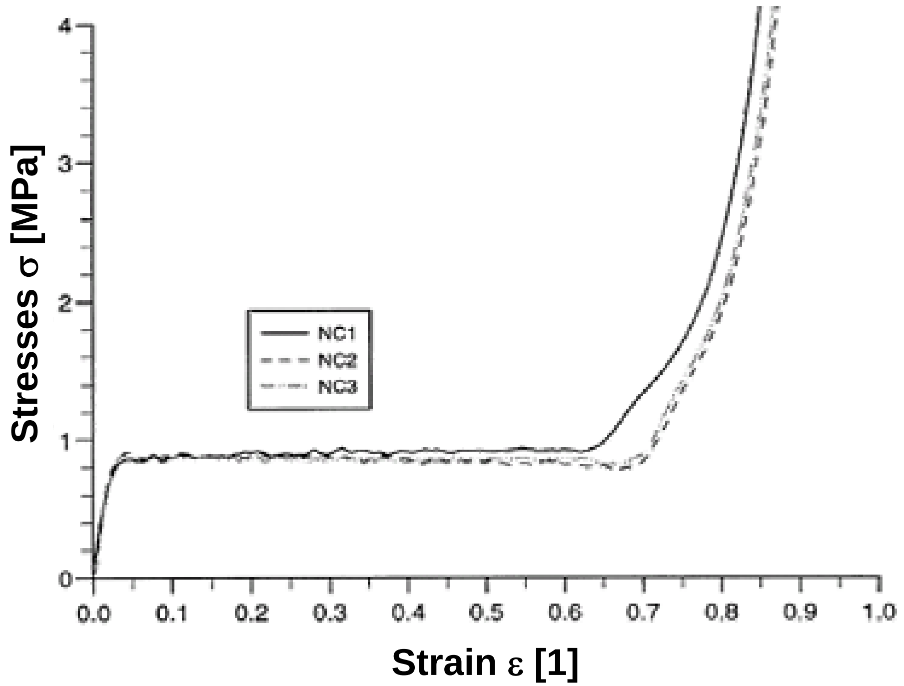
Test Facility: Tension-/Compression-/Torsion-Testmachine

Analysis of Test Results

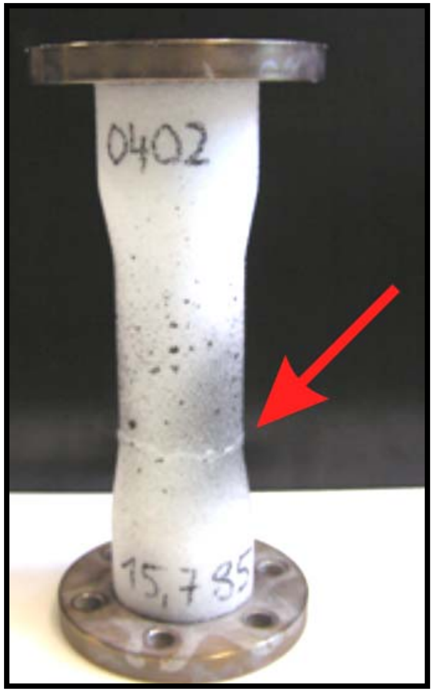
- Compression Test:

  Stress at the first Collapse of the Cells is used for Compression Strength

Stress-Strain-Diagram



Failure Mode



- Tension and Shear Test:

  Stresses at Breackage Point is used for Tensile and Shear Strength Properties

3-D Failure Criterion Model for ROHACELL®: General Equations

Stress Matrix:

$$\underline{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}$$

Equivalent Stress σ_V of the 3D Failure Criterion Model Φ_2:

$$\sigma_V = \frac{\sqrt{(12\,a_2 + 12\,a_1 + 12)\,I_2 + \left(4\,a_2^2 + (4\,a_1 + 4)\,a_2 + a_1^2\right) I_1^2} + a_1 I_1}{2\,a_2 + 2\,a_1 + 2}$$

with: **1. Invariant:** $I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}$

2. Invariant: $I_2 = \frac{1}{3}\left[\sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2 - \sigma_{11}\,\sigma_{22} - \sigma_{11}\,\sigma_{33} - \sigma_{22}\,\sigma_{33} + 3\,(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2)\right]$

$$a_1 = \frac{k^2\,(d - 1)}{d}$$

$$a_2 = \frac{k^2}{d} - 1$$

$$d = \frac{R^-_{11}}{R^+_{11}}$$

$$k = \sqrt{3}\,\frac{R_{12}}{R^+_{11}}$$

Remarks:

- R^-_{11} = **Compressive Strength of ROHACELL® Grade and Type**
- R^+_{11} = **Tensile Strength of ROHACELL® Grade and Type**
- R_{12} = **Shear Strength of ROHACELL® Grade and Type**

Effort:

$$A = \frac{\sigma_V}{R^+_{11}}$$

Factor of Safety:

$$RF = \frac{1}{A}$$

Margin of Safety:

$$MS = RF - 1$$

3-D Failure Criterion Model: ROHACELL® WF

ROHACELL® Type/ Grade	51 WF	110 WF	200 WF
Density [kg/m^3]	50,4	102,8	194,8
Parameter a_1 [1]	-0,31	0,07	0,45
Parameter a_2 [1]	0,00077	0,02	0,53
Tensile Strength [MPa]	1,3	3,7	5,27
Compressive Strength [MPa]	0,9	3,9	6,8
Shear Strength [MPa]	0,6	2,2	4,3
Storage Condition: Temperature [°C] rel. Humidity [%]	23 50	23 50	23 50
Test Condition: Temperature [°C] rel. Humidity [%]	23 50	23 50	23 50

3-D Failure Criterion for ROHACELL® 51 WF in σ_{11}/τ_{12}-Plane

3-D Failure Criterion Model:

$$\Phi_2 = \frac{3 \cdot I_2' + a_1 \cdot \sigma_V \cdot I_1 + a_2 \cdot I_1^2}{1 + a_1 + a_2} = \sigma_V^2 < \sigma_G^2$$

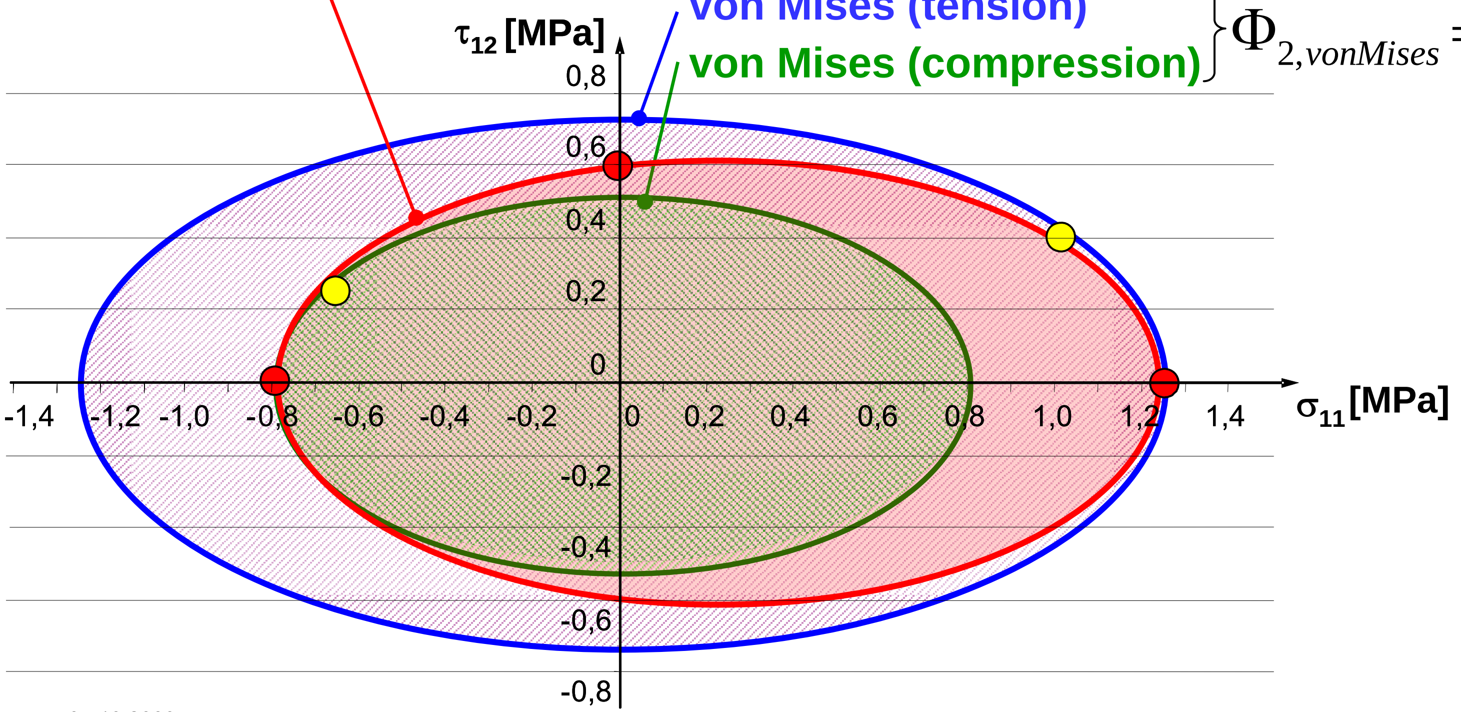
ϕ_2 Model consider:

- Compressible Material Behaviour of ROHACELL®
- Strength Differential Effect of ROHACELL®, i. e. tension strength ≠ compressive strength ≠ shear strength

von Mises (tension)

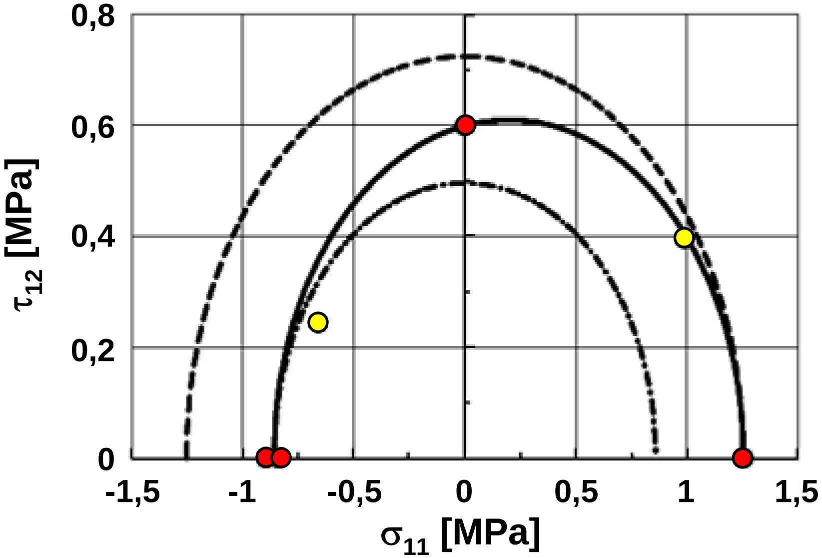
von Mises (compression)

$$\Phi_{2,vonMises} = 3 \cdot I_2' = \sigma_V^2 < \sigma_G^2$$

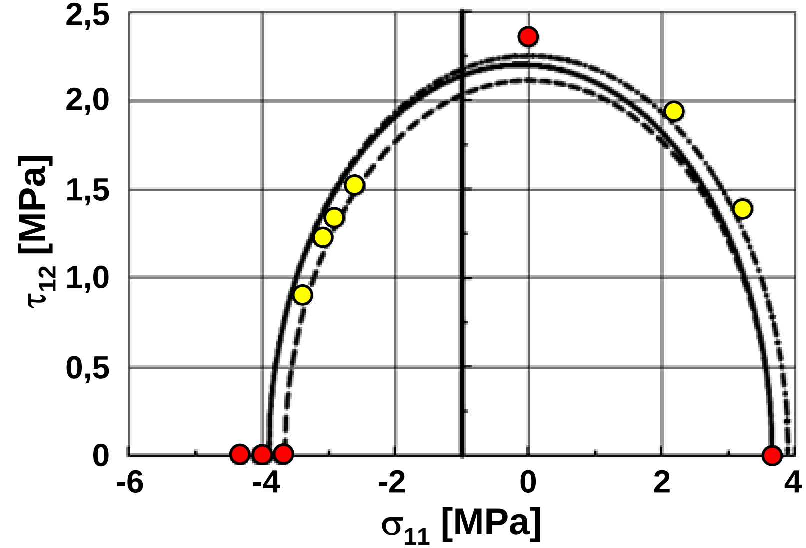


3-D Failure Criterion for ROHACELL® 51, 110 and 200 WF in σ_{11}/τ_{12}-Plane

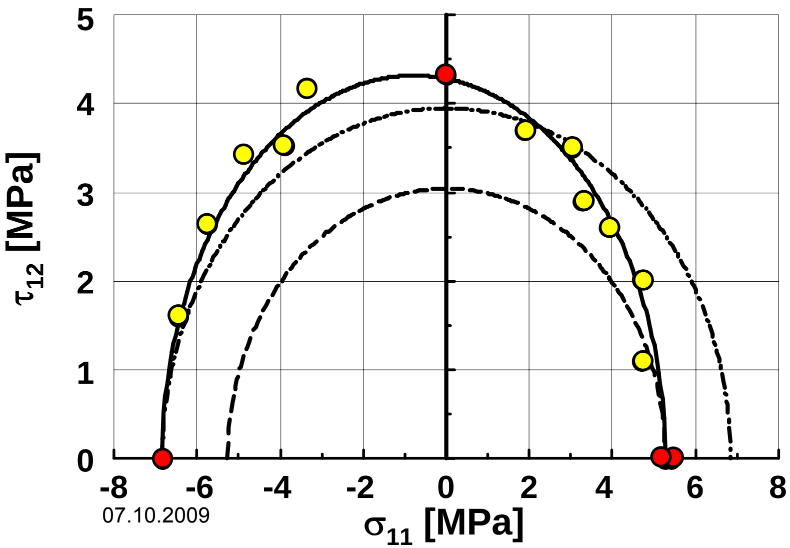
ROHACELL® 51 WF



ROHACELL® 110 WF



ROHACELL® 200 WF



Legend:

3-D Failure Criterion Φ_2

$$\Phi_2 = \frac{3 \cdot I_2' + a_1 \cdot \sigma_V \cdot I_1 + a_2 \cdot I_1^2}{1 + a_1 + a_2} = \sigma_V^2 < \sigma_G^2$$

Potential Body according to von Mises (Tension)

Potential Body according to von Mises (Compression)

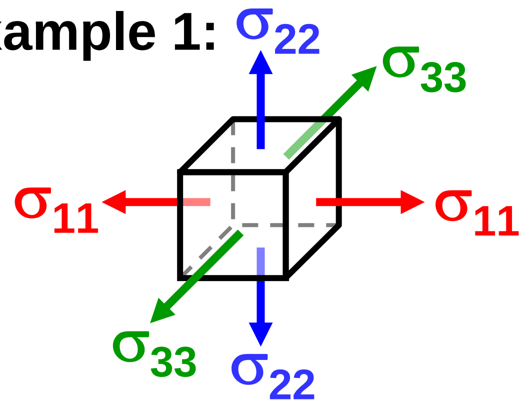
$$3 \cdot I_2' = \sigma_V^2 < \sigma_G^2$$

Experimental Data for Determination of Φ_2

Experimental Data for Validation of Φ_2

Example 1: 3-D Failure Criterion Φ_2 in comparison to other Failure Criteria

Example 1:



σ_{11}, σ_{22}, σ_{33} = 1,25 MPa = R^{+}_{11} (uniaxial Tensile Strength)

und

σ_{12} , σ_{13}, σ_{23} = 0

Strength Hypothesis:

Normal Stress Hypothesis (NSH):
A_{NSH} = 1,00

Shear Stress Hypothesis according to Tresca:
A_{Tresca} = 0

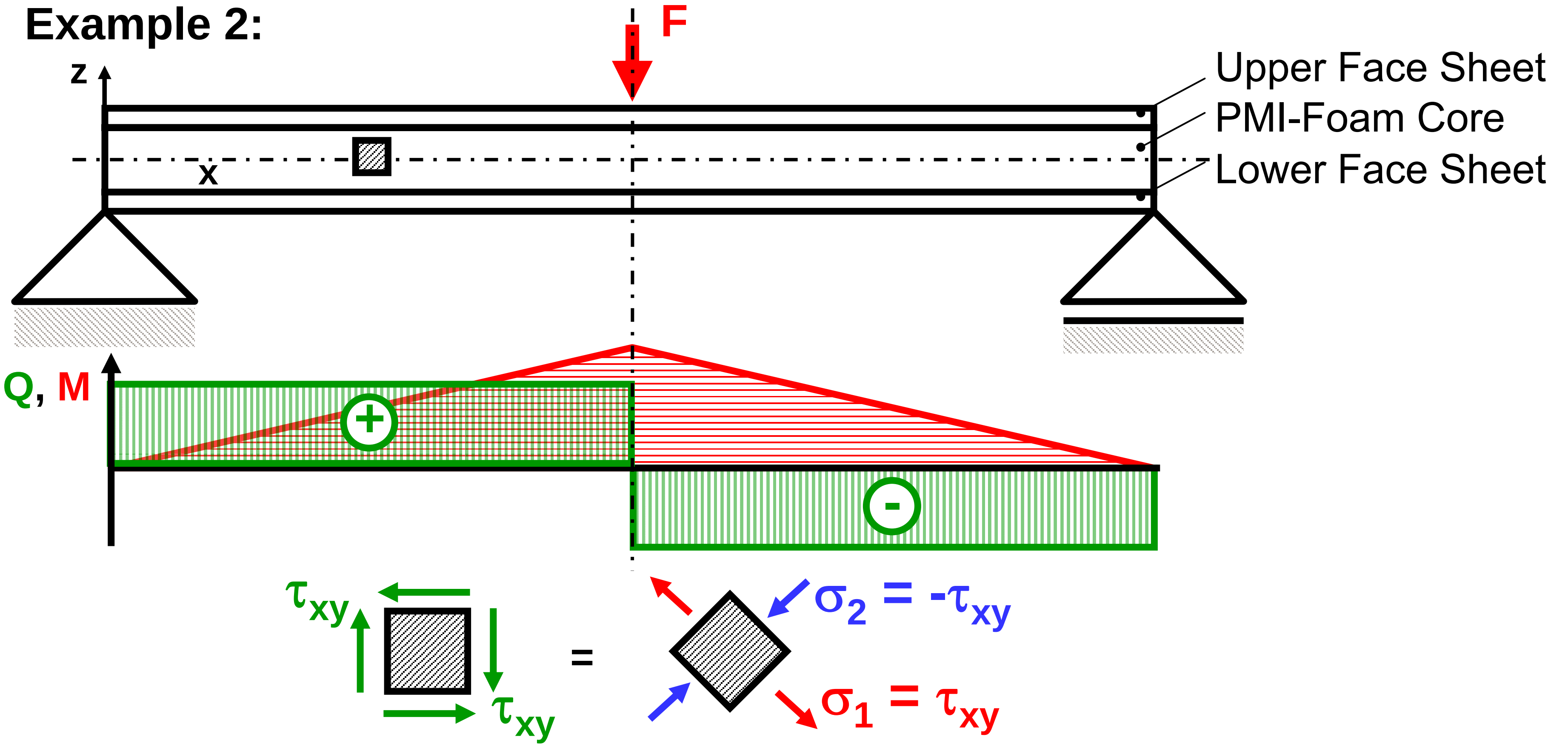
Hypothesis of strain energy of distortion according to von Mises:
$A_{von\ Mises}$ = 0

3D-Failure Model according to Φ_2-Potential:
$A_{\Phi 2}$ = 1,43 ($A_{\Phi 2}$ = 1 bei σ_{11}, σ_{22}, σ_{33} = 1,13 MPa)

e. g. ROHACELL® 51 WF

Example 2: 3-D Failure Criterion Φ_2 in comparison to other Failure Criteria

Example 2:



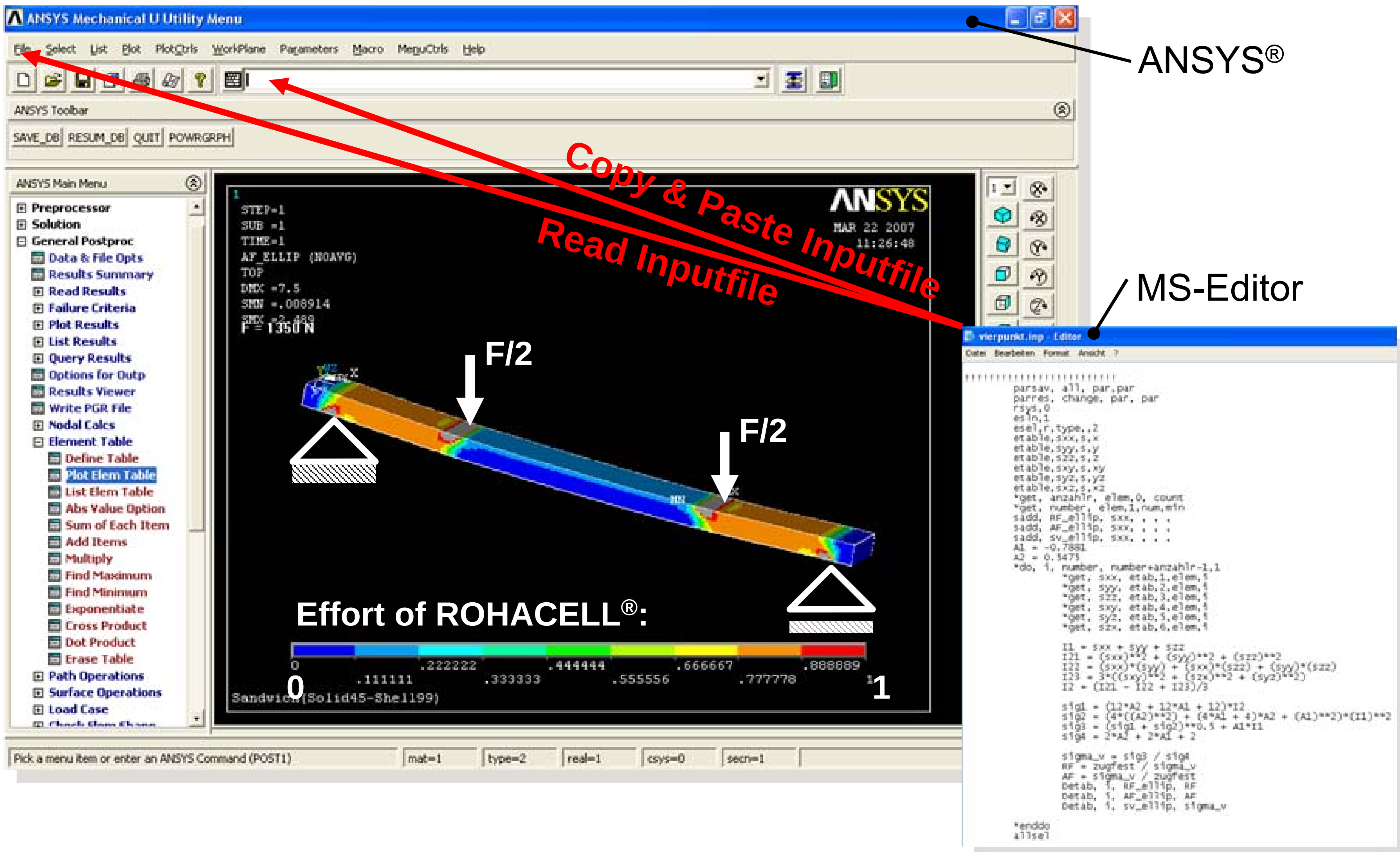
$\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{13}, \sigma_{23} = 0$ und $\sigma_{12} = \tau_{xy} = R_{12} = 0{,}6$ MPa

$\mathbf{A_{\Phi 2} = 1}$ (vgl. $A_{\text{von Mises}} = 0{,}83$, $A_{NSH} = 0{,}48$, $A_{\text{Tresca}} = 0{,}96$)

e. g. ROHACELL® 51 WF

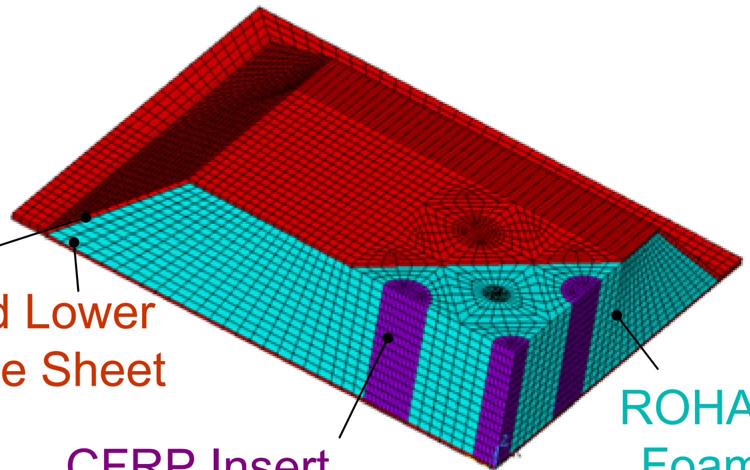
Implementation of 3-D Failure Criterion Φ_2 in FE-Programs (e. g. ANSYS®)

EVONIK INDUSTRIES

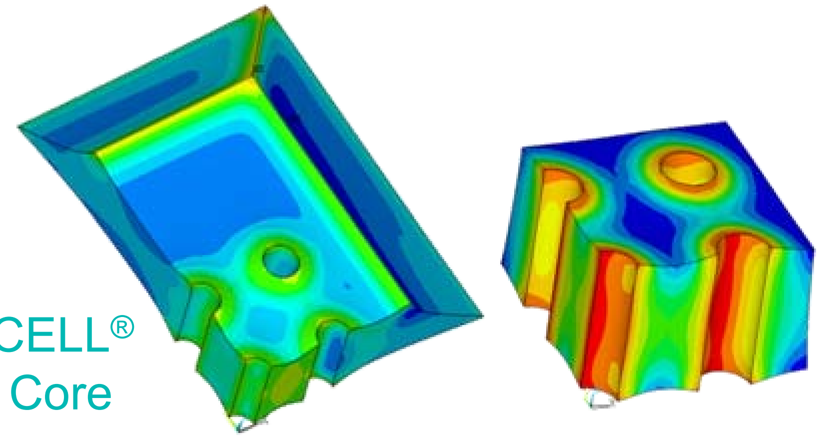


Implementation of 3-D Failure Criterion Φ_2 in Microsoft EXCEL

FE Model



Stress Distribution in the Core Material



$$\sigma_V = \frac{\sqrt{(12\,a_2 + 12\,a_1 + 12)\,I_2 + \left(4\,a_2^2 + (4\,a_1 + 4)\,a_2 + a_1^2\right)I_1^2} + a_1 I_1}{2\,a_2 + 2\,a_1 + 2} \qquad A = \frac{\sigma_V}{R_{11}^+}$$

ROHACELL® Grade	RIST											
ROHACELL® Type	71											
Tensile Strength R^+_{11} [MPa]:	2,135	(positive value)										
Compressive Strength R^+_{11} [MPa]:	-1,345	(negative value)										
Shear Strength R_{12} [MPa]:	1,100	(positive value)										
Parameter d:	0,62998											
Parameter k:	0,89239		Stress Components [MPa]						Invariant		Equivalent Stress [MPa]	Effort A [1
Parameter a_1:	-0,468		σ_{11}	σ_{22}	σ_{33}	τ_{12}	τ_{13}	τ_{23}	I_1	I_2	σ_V	A
Parameter a_2:	0,264		1,200	1,200	1,900	0,400	0,200	0,200	4,300	0,4033	1,778	0,833

Material Effort: A = 0,833 Factor of Safety: RF = 1,2 Margin of Safety: MS = 0,2

Summary/ Conclusion

- **Development of a new 3-D Failure Criterion Φ_2 to determine the Effort (A), Safety Factor (RF) or Safety of Margin (MS) for PMI-Foams ROHACELL®**
  3-D Failure Criteria Φ_2 consider:
  - Compressible Material Behaviour
  - Strength Differential Effect
- **3 Parameters are needed to describe the surface of the Breackage in the 3-D Stress Space. These 3 Parameters can be determined by using the Test Results of pure Tension, Compression and Shear Loadings.**
- **Combined Tension/Shear and Compression/Shear Loadings were carried out to validate the 3-D Failure Criteria**
- **3-D Failure Criteria Φ_2 is already implemented in ANSYS® (Röhm), ABAQUS® (DKI) and Patran/Nastran® (Airbus)**
- **The new 3-D Failure Criteria is more suitable for describing the Material Behaviour of ROHACELL® than the Potential Body according to von Mises and other Failure Criteria.**
- **Due to the current Results, use of the 3-D Failure Criterion to determine the Material Effort of ROHACELL® is advisable.**